ESI Workshop: Free Boundary Problems and related Evolution Equations

Erwin-Schrödinger Institute

February 21-25, 2022

Book of Abstracts

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last updated on **February 21, 2022**

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On the convergence of critical points of the Ambrosio-Tortorelli functional

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The numerical approximation of the Mumford-Shah functional, $u \mapsto MS(u)$, is a difficult question related to the fact that the jump set J_u of the unknown u is a free discontinuity. A possible approach is the use of a phase field method which consists in replacing J_u by a smooth phase field variable v equal to 1 everywhere in the domain, but on a small ε-neighborhood of J_u, where it vanishes. The variational approximation of the Mumford-Shah functional by such ε-regularized functionals has been obtained by Ambrosio & Tortorelli in [1]. As a byproduct, the convergence of global minimizers becomes an immediate consequence of the fundamental theorem on Γ-convergence.

Unfortunately, the Ambrosio-Tortorelli functional $(u, v) \mapsto AT_\varepsilon(u, v)$ fails to be convex with respect to the pair (u, v) (while it is separately convex) and it leads to serious issues regarding the numerical approximation of its global minimizers (which however do exist). A possible remedy consists in performing an alternate minimization algorithm [3]. Limit points of such an algorithm only turn out to be critical points of the Ambrosio-Tortorelli functional, which have no reasons to be global minimizers. The question of convergence of critical points becomes therefore natural. In the one-dimensional case, this problem has been addressed in [4, 5].

We extend here these results to any arbitrary dimension by proving that any critical points $(u_\varepsilon, v_\varepsilon)$ of AT_ε converge to a critical point u of MS (in the sense of inner variations) upon the additional assumption of the convergence of the energies, i.e. $AT_\varepsilon(u_\varepsilon, v_\varepsilon) \to MS(u)$. As a byproduct, we also obtain the convergence of the second inner variation, which implies that stable critical points of AT_ε converge to stable critical points of MS. The proof rests on elliptic PDEs and geometric measure theoretic arguments. We first establish a regularity theory in Hölder spaces for the system of elliptic partial differential equations satisfied by critical points of the Ambrosio-Tortorelli functional. It allows us to derive the first inner variations of AT_ε which shows a varifold structure. Then, we characterize the limit varifold as the rectifiable varifold associated to J_u.

This is a joint work with Vincent Millot (Université Paris-Est) and Rémy Rodiac (Université Paris Saclay).

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Rigidity results for measurable sets

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Let $\Omega \subset R^d$ be a set with finite Lebesgue measure such that, for a fixed radius $r > 0$, the Lebesgue measure of $\Omega \cap B_r(x)$ is equal to a positive constant when x varies in the essential boundary of Ω. We prove that Ω is a ball (or a finite union of equal balls) provided it satisfies a nondegeneracy condition, which holds in particular for any set of diameter larger than r which is either open and connected, or of finite perimeter and indecomposable. This is a joint work with Ilaria Fragalà.

An obstacle problem for the p-elastic energy

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The p-elastic energy of a curve $f : I \to \mathbb{R}^2$, is given by

$$\mathcal{E}_p(f) = \int_I |\vec{\kappa}|^p \, ds \,,$$

with $\vec{\kappa}$ the curvature of f and s the arc-length parameter. Here $p \in (1, \infty)$ and for $p = 2$ one recovers the Bernoulli model of an elastic rod.

In this talk we consider the obstacle problem obtained minimizing the p-elastic energy on graphs constrained to stay above a given obstacle. We discuss existence, uniqueness and symmetry of minimizers. The main question we address in the talk is: Which is the main cause of the loss of regularity of minimizers, the presence of obstacle or the degeneracy of the Euler-Lagrange equation?

This is joint work with Marius Müller (Freiburg University), Shinya Okabe (Tohoku University) and Kensuke Yoshizawa (Tohoku University).

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A proof of Taylor scaling for curvature-driven dislocation motion through random arrays of obstacles

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We prove Taylor scaling (in some contexts also known as Friedel scaling) for dislocation lines characterized by line-tension and moving by curvature under the action of an applied shear stress in a plane containing a random array of obstacles. Specifically, we show—in the sense of optimal scaling—that the critical applied shear stress for yielding, or percolation-like unbounded motion of the dislocation, scales in proportion to the square root of the obstacle density. For sufficiently small obstacle densities, Taylor scaling dominates the linear-scaling that results from purely energetic considerations as well as the two-thirds Labusch scaling, and, therefore, characterizes the dominant rate-limiting mechanism in that regime.

Joint work with L. Courte (Luxembourg) and M. Ortiz (Bonn/Pasadena).

Phase Separation in Heterogeneous Media

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A variational model in the context of the gradient theory for fluid-fluid phase transitions with small scale heterogeneities is studied. In the case where the scale of the small homogeneities is of the same order of the scale governing the phase transition, the interaction between homogenization and the phase transitions process leads to an anisotropic interfacial energy. Bounds on the homogenized surface tension are established. In addition, a characterization of the large-scale limiting behavior of viscosity solutions to non-degenerate and periodic Eikonal equations in half-spaces is given.

This is joint work with Riccardo Cristoferi (Radboud University, The Netherlands), Adrian Hagerty, Cristina Popovici, Rustum Choksi (McGill), Jessica Lin (McGill), and Raghavendra Venkatraman (NYU).

Fracture with healing as a template for cavitation

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In this joint work with Alessandro Giacomini (Brescia) and Oscar Lopez-Pamiés (Urbana Champaign) we propose a simple model of healing for brittle fracture as a first step towards cavitation. The idea is that healing will also be brittle and that there is a competition between add-cracks and subtract-cracks. We present an existence theorem in a 2d topological setting, the only available framework as of yet.

The isoperimetric inequality outside a convex set: the case of equality

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In 2007 Choe, Ghomi and Ritoré proved the relative isoperimetric inequality outside a convex set C. More precisely they showed that if $E \subset \mathbb{R}^N \setminus C$ has finite mass m, then the perimeter of E outside C is equal to the surface measure of the half sphere enclosing the half ball with mass m. Moreover, they showed that if C is smooth and the equality holds in this isoperimetric inequality, then E is a half ball sitting on C. In this talk I will discuss the equality case when C is any convex set, not necessarily smooth. This is a joint work with M. Morini.

On the interaction of mean curvature flow and diffusion on evolving hypersurfaces

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We consider a geometric problem consisting of an evolution equation for a closed hypersurface coupled to a parabolic equation on this evolving surface. More precisely, the evolution of the hypersurface is determined by a scaled mean curvature flow that depends on a quantity defined on the surface via a diffusion equation. This system arises as a gradient flow of a simple energy functional. Assuming suitable parabolicity conditions, we derive short-time existence for the system. The proof is based on linearization and a contraction argument. For this, we parameterize the hypersurface via a height function and thus the system, originally defined on an evolving surface, can be transformed onto a fixed reference surface. The result is formulated in a classical sense, holds for the case of embedded and immersed hypersurfaces alike and provides an existence time independent of small changes in the initial surface. Afterwards, several properties of the solution are analyzed. Emphasis is placed on to what extent the surface in our setting evolves the same as for the usual mean curvature flow. To this end, we show that the surface area is strictly decreasing but give an example of a surface that exists for infinite times nevertheless. Moreover, mean convexity is conserved whereas convexity is not. Finally, we construct an embedded hypersurface that develops a self-intersection in the course of time.

Finally, we discuss how solutions can be computed numerically with the help of an evolving surface finite element discretization. We will discuss optimal error bounds and present numerical experiments illustrating the above discussed qualitative properties of the flow as well as the convergence behaviour.

The analytical part is a joint work with Helmut Abels and Felicitas Bürger (both University Regensburg) and the numerical part is joint work with Charlie Elliott (University of Warwick) and Balázs Kovács (University Regensburg).

Derivation of surface tension of grain boundaries in polycystals

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Inspired by a recent result of Lauteri and Luckhaus, with derive, via Gamma convergence, a surface tension model for polycrystals in dimension two. The starting point is a semi-discrete model accounting for the possibility of having crystal defects. The presence of defects is modelled by incompatible strain field with quantised curl. In the limit as the lattice spacing tends to zero we obtain an energy for grain boundaries that depends on the relative angle of the orientations of the two neighbouring grains. The energy density is defined through an asymptotic cell problem formula. By means of the bounds obtained by Lauteri and Luckhaus we also show that the energy density exhibits a logarithmic behaviour for small angle grain boundaries in agreement with the classical Shockley Read formula.

The talk is based on a paper in preparation in collaboration with Emanuele Spadaro.

On a singular limit of a single-well Modica-Mortola functional and its applications

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It is important to describe the motion of multi-phase boundaries. One typical way is to use the Kobayashi-Warren-Cater energy. It is a weighted total variation with single Modica-Mortola energy. Modica-Mortola energy is very popular especially with double-well potential to handle two-phase problems. It is important to characterize a singular limit of such a type of energies as the thickness parameter of a diffuse interface tends to zero. In the case of double-well potentials, such a problem is well studied and it is formulated, for example, as the Gamma limit under L^1 convergence.

However, if one considers the single-well Modica-Mortola functional, it turns out that L^1 convergence is too rough even in the one-dimensional problem.

We characterize the Gamma limit of a single-well Modica-Mortola functional under the topology which is finer than L^1 topology. In a one-dimensional case, we take the graph convergence. In higher-dimensional cases, it is more involved. As an application, we give an explicit representation of a singular limit of the Kobayashi-Warren-Carter energy. Since the higher-dimensional cases can be reduced to the one-dimensional case by a slicing argument, studying the one-dimensional case is very fundamental. A key idea to study the one-dimensional case is to introduce "an unfolding of a function" by changing an independent variable by the arc-length parameter of its graph. This is based on joint work with Jun Okamoto (The University of Tokyo), Masaaki Uesaka (The University of Tokyo, Arithmer Inc.), and Koya Sakakibara (Okayama University of Science, RIKEN). Results of the one-dimensional case have been published in [1] while the multi-dimensional case is work in progress.

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Cahn-Hilliard-Hele-Shaw systems with singular potential

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In a two-dimensional Hele-Shaw cell, provided that the viscous forces dominate the inertial ones, the well-known Navier-Stokes-Cahn-Hilliard system for an incompressible binary flow can be approximated by the so-called Cahn-Hilliard-Hele-Shaw (CHHS) (or Cahn-Hilliard-Darcy) system (see [9]). In three dimensions, CHHS systems are used to describe fluid flow in a porous medium and in solid tumor growth modeling through diffuse interfaces (see, for instance, [2, 3, 5, 6] and references therein). I intend to present and compare a number of recent results (see [1, 4, 7, 8]), focusing, in particular, on uniqueness and regularity issues. All of them are characterized by a mixing entropy term which is not approximated, as usual, by a polynomial.

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Relaxation of functionals with linear growth: Interaction of measures and free discontinuities

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For an integral functional defined on functions (u, v) in $W^{1,1} \times L^1$ featuring a strong interaction term between u and v, we calculate its relaxation in the space of functions with bounded variations and Radon measures. Emerging free discontinuities can interact with the measure variable, which can lead to non-trivial behavior of minimizing sequences. As it turns out, these can exhibit interesting oscillatory and concentrating behavior even if the leading terms in the functional (in the sense of highest order derivatives) are convex and coercive.

Joint work: Martin Kružík (Prague), Elvira Zappale (Rome)

Equilibrium for Multiphase Solids with Eulerian Interfaces

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We describe a general phase-field model for hyperelastic multiphase materials. We can consider shape memory alloys as an example of such materials. The model features an elastic energy functional that depends on the phase-field variable and a surface energy term that depends in turn on the elastic deformation, as it measures inter- faces in the deformed configuration. We prove existence of energy minimizing equilibrium states and Γ-convergence of diffuse-interface approximations to the sharp-interface limit.

It is a joint work with D. Grandi (Ferrara), E. Mainini (Genoa), and U. Stefanelli (Vienna).

A new varifold solution concept for mean curvature flow: Convergence of the AllenâCahn equation and weak-strong uniqueness

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Mean curvature flow is one of the most fundamental geometric evolution equations and appears in many surface-tension driven problems. Although the equation has an instantaneous smoothing effect, generically, singularities appear in finite time. One is led to consider weak solutions which persist through these singular events. Folklore says that mean curvature flow is a gradient flow with the caveat that the underlying metric is completely degenerate. In this talk, after discussing previous weak solution concepts, I will present a new notion which has its roots in the theory of gradient flows and relies on basic geometric measure theory. I will show that these solutions arise naturally in the sharp-interface limit of the Allen-Cahn equation and in addition satisfy a weak-strong uniqueness principle. The latter property is a fundamental difference to well-known Brakke solutions, which a priori may disappear at any given time and are therefore fatally non-unique.

This is joint work with Sebastian Hensel (U Bonn).

The prescribed mean curvature measure equation

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Finding graphs of prescribed mean curvature is one of the classical problems of Calculus of Variations, strongly motivated by capillarity theory. We present a novel, weak formulation of the equation

$$\operatorname{div} \frac{\nabla u}{\sqrt{1+|\nabla u|^2}} = \mu$$

where the prescribed mean curvature μ can be a signed measure, and for which existence of solutions can be proved under minimal assumptions on μ. The existence proof is obtained by minimizing a suitable functional on a class of BV functions with Dirichlet boundary conditions. It combines various tools like the notion of λ-*pairing*, a refined form of Anzellotti-Giaquinta theorem in BV, and a convex duality argument. This is a joint collaboration with Giovanni Comi.

Explicit minimizers for a class of nonlocal interaction energies

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Nonlocal interaction energies are continuum models for large systems of particles, where each particle interacts not only with its immediate neighbors, but also with particles that are far away. These energies arise in many different applications, such as biology (population dynamics), physics (Ginzburg-Landau vortices), and material science (dislocation theory).

A fundamental question is the understanding of the optimal arrangement of particles at equilibrium, which are described, at least in average, by minimizers of these energies.

In this talk I will focus on a class of nonlocal energies that are perturbations of the Coulomb energy and I will show an explicit characterization of their minimizers.

This is based on joint works with J. Mateu, L. Rondi, L. Scardia, and J. Verdera.

The asymptotics of the area preserving mean curvature flow in two dimensions

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We provide the first general result for the asymptotics of the area preserving mean curvature flow in two dimensions showing that flat flow solutions starting from any bounded set of finite perimeter converge to a finite union of equally sized disjoint disks with exponential rate.

Isoperimetric clusters

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We discuss the existence of generalized clusters with a finite or infinite number of components, having minimal total perimeter and given masses, in metric measure spaces homogeneous with respect to a group acting by measure preserving homeomorphisms. Such clusters are a natural relaxation of a cluster and can be thought of as "albums" with possibly infinite pages, the total perimeter being calculated by summation over all pages, and being minimal among all generalized clusters with the same masses.

About soap films spanning a nonregular tetrahedron

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The question of whether the cone is the minimal surface spanning the edges of a regular tetrahedron is not completely settled. It is believed that to be the case, but there is no rigorous mathematical proof. Indeed, finding minimal surfaces (and by "minimal surface" I think of surfaces that can be attained as "real" soap films that span some given frame) is a "slippery" subject (sorry for the pun). The problem needs to be clearly stated, however there are many different ways to do that, each one with advantages and drawbacks. Just to cite a few: Reifenberg, Taylor, De Giorgi, Almgren, Morgan, Harrison. Indeed it seems that there is not a single approach that is satisfactory in all different contexts. One possible approach, first introduced by Ken Brakke [1], consists in manually designing a suitable covering space for the complement of the frame. Different covering spaces can lead to different absolute minimizers, which can or cannot be desirable.

This approach allows to obtain interesting solutions, also solutions that can exhibit "partial wetting", a phenomenon that seems not yet much investigated.

Going back to the tetrahedral frame, we show that an appropriately designed triple covering space strongly suggests the existence of an absolute minimizer that exhibits a nontrivial topology. This is obtained at the expense of deforming the tetrahedron into an elongated version. Of course this example by no means settles the question of the minimality of the cone for the regular tetrahedral frame.

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Viscoelasticity with Time-Dependent Memory Kernels

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We consider the model equation arising in the theory of viscoelasticity

$$\partial_{tt}u(t) - \big[1 + k_t(0)\big]\Delta u(t) - \int_0^\infty k_t'(s)\Delta u(t-s)ds + f(u(t)) = g$$

in the presence of a (convex, nonnegative and summable) memory kernel $k_t(\cdot)$ explicitly depending on time. Such a model is apt to describe, for instance, the dynamics of aging viscoelastic materials. From the mathematical viewpoint, this translates into the study of dynamical systems acting on time-dependent spaces, according to the recently established theory of Di Plinio *et al.* [3]. After giving the proper notion of solution, along with a global well-posedness result, we focus on the asymptotic properties of the solutions, proving the existence and the regularity of the time-dependent global attractor for the dynamical process generated by the equation. In addition, when k_t approaches a multiple $m\delta_0$ of the Dirac mass at zero as $t \to \infty$, we show that the asymptotic dynamics of our problem is close to the one of its formal limit

$$\partial_{tt}u(t) - \Delta u(t) - m\Delta\partial_t u(t) + f(u(t)) = g$$

describing viscoelastic solids of Kelvin-Voigt type. This talk is based on the papers [1, 2].

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Resolution of singularities of the network flow

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The curve shortening flow is an evolution equation in which a curve moves with normal velocity equal to its curvature (at any point and time) and can be interpreted as the gradient flow of the length. We consider the same flow for networks (finite unions of sufficiently smooth curves whose end points meet at junctions). Because of the variational nature of the problem, one expects that for almost all the times the evolving network will possess only triple junctions where the unit tangent vectors forms angles of 120 degrees (regular junctions). However, even if the initial network has only regular junctions, this property is not preserved by the flow and junctions of four or more curves may appear during the evolution. The aim of this talk is first to describe the process of singularity formation and then to explain the resolution of such singularities and how to continue the flow in a classical PDE framework.

This is a research in collaboration with Jorge Lira (Universidade Federal do Ceará), Rafe Mazzeo (Stanford University) and Mariel Sáez (Pontificia Universidad Católica de Chile).

Stability results for nonlocal geometric evolutions

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We introduce a notion of uniform convergence for local and nonlocal curvatures and we propose an abstract method to prove the convergence of the corresponding geometric flows, within the level set formulation.

We apply such a general theory to characterize the limits of several nonlocal geometric evolutions. We study the limit of the s-fractional mean curvature flows as $s \to 0^+$ and $s \to 1^-$. Moreover, in analogy with s-fractional mean curvature flows, we introduce the notion of s-Riesz curvature flows and characterize its limit as $s \to 0^-$. Furthermore, using a suitable core-radius regularization, we define s-fractional perimeters and s-fractional curvatures also for $s \geq 1$ and we show that - as the core-radius tends to 0 - the corresponding geometric flows converge to the classical mean curvature flow. Finally, we discuss also the stability of the s-fractional heat flows as $s \to 0^+$ and $s \to 1^-$.

The results discussed here are obtained in collaboration with A. Cesaroni (Padova), V. Crismale (Sapienza, University of Rome), L. De Luca (CNR, Rome), A. Kubin (TU Munich), A. Ninno (Sapienza, University of Rome), M. Novaga (Pisa).

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Dynamic boundary conditions as a limit of a boundary layer problem

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In our earlier work [2] we have studied the total variation flow whose special feature was that the motion of facets was determined by the dynamic boundary conditions (DBC). We want to derive the DBC as a limit of the boundary layer problem. We present our analysis for the heat equation, because it is a bit simpler that in the case of the TV flow. We will address this problem for strong and weak solutions.

The limit passage in the case of strong solutions in based on the de Giorgi theory of convergence of gradient flows. It is also interesting to notice that the approximate problem is considered in $L^2(\Omega)$, while the limit system is studied in $L^2(\Omega) \times L^2(\partial\Omega)$.

An important technical device is a version of Reilly identity, see [3], which is of independent interest.

The presentation is based on a joint paper with Y.Giga and M.Åasica, [1]

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A non-parametric Plateau problem with partial free boundary

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We discuss a Plateau problem in non-parametric form with a partial free boundary. We show existence and regularity of solutions, and then we compare the obtained solutions with the solutions of a parametric plateau problem with corresponding boundary data.

Extensions of the Cahn-Hilliard equation to a temperature-dependent setting

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In this talk we consider the dynamics of binary media that can be modeled by the Cahn-Hilliard equation. We investigate the situation that the phase separation of the binary media depends on temperature. Firstly, we derive models by combining ideas from non-equilibrium thermodynamics with an energetic variational approach. Secondly, we discuss selection criteria for these models and provide some analytical results regarding well-posedness of one of the models.

This is joint work with Francesco De Anna, Chun Liu and Jan-Eric Sulzbach.

High-contrast random composites: homogenisation framework and new spectral phenomena

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We study the homogenisation problem for elliptic high-contrast operators A_ε whose coefficients degenerate as $\varepsilon \to 0$ on a set of randomly distributed inclusions. We discuss the limit operator (in the sense of the resolvent convergence) and the convergence of spectrum. On the bounded domain the limiting spectrum is equal to the spectrum of the limit operator, while in the whole space setting the spectrum of the limit operator is the subset of the limiting spectrum. Additionally we characterize the limiting spectrum in the case of finite correlation. This is a joint work with Mikhail Cherdantsev (University of Cardiff) and Kirill Cherednichenko (University of Bath).

References

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